Mutual Inductance of Two Coils

Homework 10
Due 12 April 2023

Mutual inductance is the basic operating principal of the transformer, motors, generators and any other electrical component that interacts with another magnetic field. Mutual induction is defined as the current flowing in one coil that induces an current in an adjacent coil. An example is shown in Fig. 12, where current I_1 flowing in coil L_1 caused a current I_2. The differential equations which describes the flow of the electric

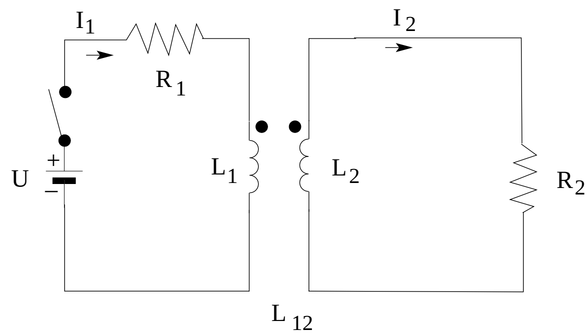

Figure 12: Mutual inductance between coils.

currents are given by

$$L_1\ \dot{I}_1(t) + R_1\ I_1(t) - L_{12}\ \dot{I}_2(t) = U(t)\,, \tag{1}$$

$$L_2\ \dot{I}_2(t) + R_2\ I_2(t) - L_{12}\ \dot{I}_1(t) = 0\,, \tag{2}$$

where $L_1 = 1.9$ H, $L_2 = 2.3$ H, $L_{12} = 0.72$ H, $R_1 = 67.5\,\Omega$, and $R_2 = 83.24\,\Omega$.[15] The voltage applied to the system varies with time according to ($U_0 = 120$ V)

$$U(t) = \begin{cases} U_0 & \text{if} \quad 0 < t \le 0.1 \text{ s}\,, \\ U_0\,\mathrm{e}^{-1.5\,t} & \text{if} \quad 0.1 \text{ s} < t \le 0.2 \text{ s}\,, \\ U_0\,\sin(-1.75\,t) & \text{if} \quad 0.2 \text{ s} < t \le 0.25 \text{ s}\,, \\ U_0\,\mathrm{e}^{-0.5\,t} & \text{if} \quad 0.25 \text{ s} < t\,. \end{cases} \tag{3}$$

Hints

To treat Eqs. (1) and (2) numerically, solve Eq. (2) for $\dot{I}_2(t)$ and substitute the result into Eq. (1). This leads for Eq. (1) to

$$\dot{I}_1(t) = [U(t) - R_1\,I_1(t) - L_{12}\,R_2\,I_2(t)/L_2]\,/\,\left[L_1 - L_{12}^2/L_2\right] \equiv S(t)\,. \tag{4}$$

Verify that this expression is correct. Next, make use of the Euler forward method to approximate the first-order time derivatives in Eqs. (4) and (2) by finite-difference expressions. This leads for Eqs. (4) and (2) to

$$I_1(t + \Delta t) = I_1(t) + S(t)\,\Delta t\,, \tag{5}$$

$$I_2(t + \Delta t) = I_2(t) + \frac{L_{12}\,S(t) - R_2\,I_2(t)}{L_2}\,\Delta t\,, \tag{6}$$

respectively.

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[15]The units are H=Ω s, F=s/Ω.

Tasks

Write a structured Fortran program which solves Eqs. (5) and (6) numerically for times $0 \leq t \leq t_{\text{final}}$, where $t_{\text{final}} = 0.50$ seconds. The initial conditions for the electric currents are $I_1(0) = I_2(0) = 0$ A. Choose $\Delta t = t_{\text{final}}/10^4$ for the temporal step size.

Program Design

- The design of the main program of your Fortran code must be as shown below:

```
program Name...

! Purpose:

  implicit none
  real :: ...

! Assign values to U_0, L1, L2, L12, R1, R2, I1_0 and I2_0 (currents at t=0),
! t_final and dt:
  call INPUT(U_0, L1, L2, L12, R1, R2, I1_0, I2_0, t_final, dt)

! Solve system of coupled differential equations and write the results for
! the electric currents I_1(t) and I_2(t) to different output files:
  call DiffEquationSolver(U_0, L1, L2, L12, R1, R2, I1_0, I2_0, t_final, dt)

end program Name...
```

- The purpose of the subroutine `INPUT` is to assign numerical values to U, L_1, L_2, L_{12}, R_1, R_2, $I_1(0)$, $I_2(0)$, t_{final}, and Δt and return them to the main program.

  The values of $I_1(0)$ and $I_2(0)$ are keyboard input. Use the `WRITE(*, '(A)', advance='NO')` format option to prompt the user to provide those values from keyboard.

  Use the `intent(out)` descriptor to declare the arguments of the subroutine.

- Equations (5) and (6) are solved for times $0 \leq t \leq t_{\text{final}}$ in the subroutine named `DiffEquationSolver`. The numerical results for the electric currents $I_1(t)$ and $I_2(t)$ are to be written to two different output files.

  Use the `intent(in)` descriptor to declare the arguments of the subroutine.

- Use a `FUNCTION` sub-program to compute the value of $S(t)$. Use the `intent(in)` descriptor to declare the arguments.

- Show the results for $I_1(t)$ and $I_2(t)$ for $0 \leq t \leq t_{\text{final}}$ graphically. Create a `pdf` file of the plot.

Submitting your Homework: Create a gzipped archive file containing your Fortran source code and `pdf` plot and email it to `ewhart317@gmail.com`. Put `PHYS 317 HW 10` in the subject line.