Photon-Electron Scattering

Homework 11
Due 19 April 2023

A photon with an energy of $E = 750$ keV hits a target electron and undergoes so-called hard Compton scattering, as shown in Fig. 7.

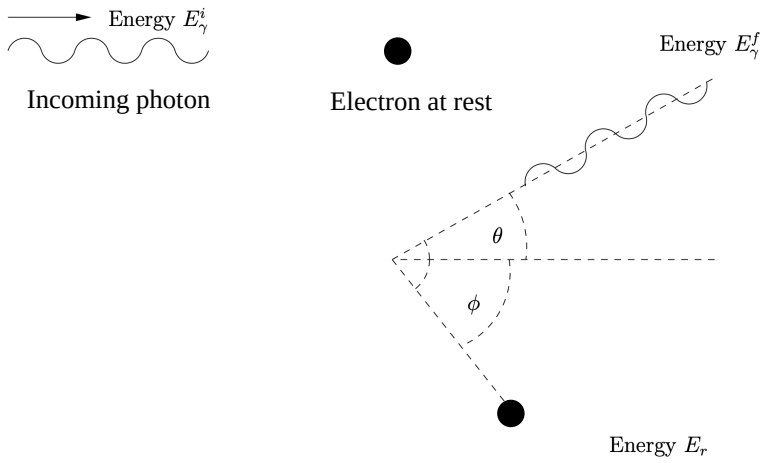

Figure 7: Schematic illustration of photon-electron scattering.

Mathematically, the scattering process is described by the following formula,

$$2\,E_\gamma^i\,E_r\sqrt{1+2E_0/E_r}\,\cos(\phi) = \left({E_\gamma^i}^2 - {E_\gamma^f}^2 + E_r^2(1+2E_0/E_r)\right)\,, \tag{1}$$

where $\phi = 67^o$ is the angle of the recoiling electron (whose energy is E_r), $E_\gamma^f = 690.3787$ keV is the energy of the scattered photon, and $E_0 = 511$ keV is the rest mass of the electron.

Task

Write a structured and well commented Fortran program which uses Newton's numerical root finding method to determine the energy E_r of the recoiling electron.

Program Design

- The value of ϕ is keyboard input. Use the `advance='no'` option in the `write` statement so that the cursor does not advance after the writing.
- Limit the maximum number of allowed iterations to 30.
- Use an initial value of $E_r = 150$ keV to start the root finding algorithm.
- Terminate the calculations if $\Delta \equiv ||E_{r(i+1)}| - |E_{r(i)}|| < 0.005$, where i denotes the i^{th} iteration step.
- A warning message is to be written to screen if the root has not been found after the maximum number of allowed iterations.

- Use a `FUNCTION` to compute the value of $Z(E_r)$ defined as

$$Z(E_r) \equiv 2\,E_\gamma^i\,E_r\sqrt{1+2E_0/E_r}\,\cos(\phi) - {E_\gamma^i}^2 + {E_\gamma^f}^2 - E_r^2(1+2E_0/E_r)\,, \tag{2}$$

  where, according to Newton's root-finding formula,

$$E_{r(i+1)} = E_{r(i)} - \frac{Z(E_{r(i)})}{Z'(E_{r(i)})}\,. \tag{3}$$

- Determine the expression for $Z'(E_r) \equiv dZ(E_r)/dE_r$ mathematically and code it up in a `FUNCTION` sub-program.
- The `intent` descriptor is to be used to declare the arguments in the `FUNCTION` sub-programs.
- For each iteration, the values of the iteration index `it`, Δ, and E_r are to be written to standard output as shown below:

```
it=            1  Delta=    40.0370331      E_r=   109.962967
it=            2  Delta=    16.9764709      E_r=   92.9864960
.
.
it=           10  Delta=        ...         E_r-    ...
```

Submitting your Homework: Email a copy of your Fortran code to `ewhart317@gmail.com`. Put `PHYS 317 HW 11` in the subject line.