

Boundary Value Problem II

Homework 13

As discussed in Worksheet 23, a particle of mass m is moving along the x axis. But this time under the action of an exponentially decaying, time-dependent force $F(t)$ given by

$$F(t)/m = 40 e^{-t} \text{ N/kg}, \quad (1)$$

where t is in seconds and x is in meter. The motion of the particle is described by the solution to the boundary value problem

$$m \frac{d^2 x}{dt^2} = F(t), \quad x(t_i = 0) = 0, \quad x(t_f) = A, \quad (2)$$

where $x(t_i = 0)$ and $x(t_f)$ denote the particle's position at the initial and final time, respectively.

Tasks

- Write a structured and well-commented Fortran program which computes the particle's path $x(t)$ for times $0 < t \leq t_f$.
- The final time is $t_f = 1$ s and the final position is $A = 10$ m.
- The position as a function of time, $x(t)$, is to be written to an external data file.
- Run the code for $n = 10$ and $n = 100$ (grid points).
- The analytic solution of Eq. (2) is given by

$$x(t) = 40 (e^{-t} - 1) + (35.7 - e^{-1}) t, \quad (3)$$

where t is in seconds and $x(t)$ is given in meters. Compute $x(t)$ from Eq. (3) for $0 \leq t \leq t_f$. The result is to be written to an external data file.

- Show the results (three curves) graphically.

Submitting your Homework: Create a gzipped archive file named `LastFirst.tgz` which contains your Fortran source code and the pdf plot and email this file to `ewhart317@gmail.com`. Put PHYS 317 HW 13 in the subject line.