Non-Linear Motion of a Physical Object

Homework 4
Due 22 February 2023

A. Review Chapter 4.3 (Numerical integration) in class textbook.

B. The position (in meter) of a certain physical object for times $0 \leq t \leq 10$ seconds is described by

$$x(t) = \pi^2 \cos^2(\alpha t)\, W(t)\,, \qquad (x(t) \text{ in meter})\,, \tag{1}$$

with $W(t)$ given by

$$W(t) = \int_0^t dt'\, \mathrm{e}^{-\alpha\sqrt{t'/5}}\, \sin^2(t')\, \cos^2(\pi\sqrt{t'})\, \sqrt{t'}\,. \tag{2}$$

The numerical value of α is 2.5.

Task

Write a structured and well-commented Fortran program which computes $x(t)$ for $0 \leq t \leq 10$ seconds. The code must contain a preamble. Use the trapezoidal rule (see chapter 4.3 in the class textbook)

$$W(t) = \int_0^t dt'\, f(t') \approx \frac{h}{2}\left(f(0) + 2\sum_{k=1}^{N-1} f(t'_k) + f(t) \right)$$

to compute the integral in Eq. (2) numerically.

Program Design

• Use a `DO` loop to compute the object's position $x(t)$ for times from $t = 0$ to $t = 10$ seconds in time steps of $\Delta t = 0.01$ seconds.
• Use a value of $N = 100$ to compute the integral $W(t)$. The value of N is keyboard input.
• Use a statement function for the integrand in Eq. (2).
• The position as a function of time is to be written to the screen (terminal). The last few lines will be as follows:

```
 t=  9.98000 seconds    x(t)=  5.38239  meter
 t=  9.99000 seconds    x(t)=  5.43078  meter
 t=  10.0000 seconds    x(t)=  5.47264  meter
```

HOMEWORK SUBMISSION INSTRUCTIONS

1. Create a sub-directory `LastFirst_HW4`.
2. Copy the source code of your Fortran program to `LastFirst_HW4`.
3. Create a gzipped archive file of your homework by typing
`tar -czvf LastFirst.tgz LastFirst_HW4/.`
4. Email the archive file `LastFirst.tgz` to `ewhart317@gmail.com`. Put `PHYS 317 HW 4` in the subject line.