Rocket Equation (I)
Apollo Moon Landing Mission

Homework 7
Due 15 March 2023

Reading Assignment: Read Sections 3.4, 3.6, and 4.2 in the Class Textbook.

The equation that describes the speed of a rocket is given by the differential equation

$$\frac{dv}{dt} = \frac{R\,u_{\rm ex}}{m_i - Rt} - g\,, \tag{1}$$

where the speed v is in m/s and time t is in seconds. This equation is called the rocket equation. The quantity $F_{\rm th} = Ru_{\rm ex}$ is the force exerted on the rocket by the exhausting fuel, and is called the thrust (R denotes the burn rate, $u_{\rm ex}$ is the speed at which the fuel is exhausted relative to the rocket). The payload of a rocket is defined as m_f/m_i, where m_f denotes the rocket's final mass, after all the fuel has been burned, and m_i is the rocket's initial mass. The quantity g denotes the gravitational acceleration (9.81 m/s^2). The Saturn–V rocket (one of the most powerful rockets ever built) used in the Apollo moon-landing program had an initial mass of $m_i = 2.85\times10^6$ kg, a payload of 27%, a burn rate of 1.384×10^4 kg/s, and a thrust of 3.4×10^7 N.

Tasks

Write a complete fortran program which solves equation (1) numerically using the Euler forward differentiation method to determine the speed and position of a Saturn–V rocket for times $0 < t \le t_b$, where $t_b = (m_i - m_f)/R$ denotes the rocket's burn time in seconds. The result for the speed will be compared with the analytical solution of the rocket equation.

Code Design

- The code must contain a preamble and must be well commented.
- The value of the temporal time step $\Delta t = 0.01$ seconds is keyboard input. The code must prompt the user to input this value from keyboard.
- The numerical values of $F_{\rm th}$, R, $u_{\rm ex}$, m_i, m_f, payload, burn time t_b, and the rocket's altitude at t_b are to be written to standard output (i.e., to the terminal).
- The results for the rocket's speed as a function of time are to be written to a data file named `speed_t.dat`
- The results for the rocket's position as a function of time are to be written to a data file named `altitude_t.dat`
- The analytical solution of the rocket equation is given by

$$v(t) = -\,u_{\rm ex}\,\ln\left(1 - \frac{R\,t}{m_i}\right) - g\,t\,. \tag{2}$$

  Extend your code to compute $v(t)$ for times $0 \le t \le t_b$ according to Eq. (2) and write the results to `speed_t_analytic.dat`.

- Create a plot that shows the rocket's altitude as a function of time.
- Create a plot that compares the numerical results for the speed of the rocket with the analytic results obtained from Eq. (2).

Submitting your Homework: Create a gzipped archive file of your homework (fortran code and the two `pdf` plots) named `LastFirst_HW7.tgz` and email this file to `ewhart317@gmail.com`. Put `LastFirst PHYS 317 HW 7` in the subject line.