An Economic Demand-and-Supply Model

Homework 8
Due 22 March 2023

In this homework, we will study a simple economic demand-and-supply model where the price $P(t)$, supply $S(t)$, and demand $D(t)$ of a commodity are represented by the differential equation

$$\frac{dP}{dt} = k(D(t) - S(t))\,, \tag{1}$$

with $D(t)$ and $S(t)$ are given by

$$D(t) = a - bP(t)\,, \tag{2}$$
$$S(t) = c(1 - \cos(\alpha t))\,, \tag{3}$$

where t is time and $k = 5.5$ is a constant. The values of a, b, c, and α are 3.67, 0.066, 2.03, and 0.77, respectively. In general, if demand exceeds supply one has $dP/dt > 0$ and the price of a commodity increases. If supply exceeds demand, $dP/dt < 0$ and the price decreases. Equation (1) can be solved analytically. The solution is given by

$$\begin{aligned} P(t) &= \left(P(0) - \frac{a-c}{b} - \frac{k^2bc}{k^2b^2+\alpha^2}\right)\mathrm{e}^{-kbt} \\ &\quad + \frac{a-c}{b} + \frac{kc}{k^2b^2+\alpha^2}\left(kb\cos(\alpha t) + \alpha\sin(\alpha t)\right)\,. \end{aligned} \tag{4}$$

Task

Write a structured and well commented Fortran program which solves the first-order differential equation (1) numerically for times $0 \le t \le T_{\max} = 40$ and the initial condition $P(0) = 1.0$.

Code Design

- The program prompts the user to input the time step value $\Delta t = 0.01$ from keyboard. Use `WRITE(*, '(A)', advance='NO')` so that the write statement will not advance to the next record (next line) after finishing the writing.
- Use the `DO WHILE` construct to compute $P(t)$, $D(t)$, and $S(t)$ for $0 \le t \le T_{\max}$.
- The results for $P(t)$, $D(t)$, and $S(t)$ are to be written to (3 different) output files.
- The code also computes $P(t)$ from Eq. (4) and writes the results to an output file.
- Create a plot which compares the numerical results for $P(t)$ with the analytic ones. (Make sure you label the graph/curves properly.)
- In a second plot, show $D(t)$ and $S(t)$ for $0 \le t \le T_{\max}$. (Make sure you label the graph/curves properly.)

Submitting your Homework: Create a gzipped archive file of your homework (Fortran source code and `pdf` plots) and email this file to `ewhart317@gmail.com`. Put `PHYS 317 HW 8` in the subject line.