Fourier Since Transforms

27 February 2023
Worksheet 11

This worksheet is for practicing the following: data output to a file, using the trapezoidal rule to numerically integrate a function, and using DO loops and statement functions.

The Fourier since transform of a function $f(t)$ is given by

$$F(\omega) = \int_0^\infty dt\ f(t)\ \sin(\omega t)\,, \tag{1}$$

where t is the time in seconds and ω denotes the angular frequency in s^{-1}. For this worksheet, the function $f(t)$ is chosen as

$$f(t) = \mathrm{e}^{-b\,t}\,, \tag{2}$$

where $b = 2\ \mathrm{s}^{-1}$. The integrand of Eq. (1) is shown graphically in Fig. 1.

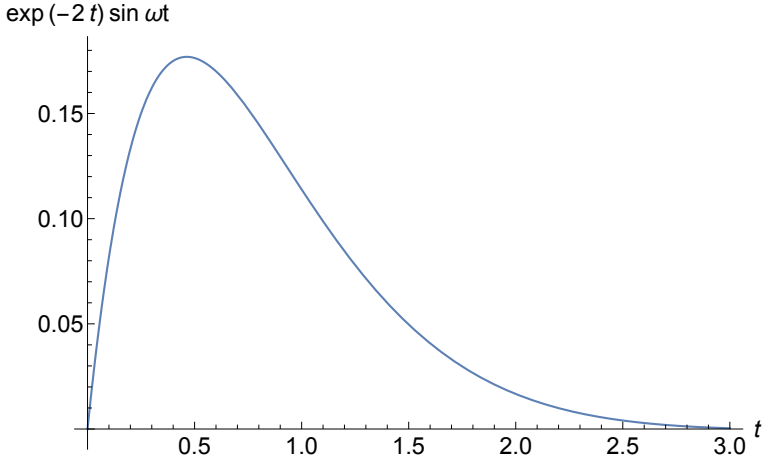

Figure 1: Graphical illustration of $\mathrm{e}^{-b\,t}\sin(\omega t)$ for $b = 2\ \mathrm{s}^{-1}$. Time t is in seconds.

Tasks

Write a structured fortran program which computes $F(\omega)$ for ω values $0 \le \omega \le 10\ \mathrm{s}^{-1}$. The numerical results are to be compared with the exact (analytic) result of the Fourier sine transform of $f(t)$ given by

$$F_b(\omega) = \frac{\omega}{\omega^2 + b^2}\,. \tag{3}$$

Code Design

- Include a short preamble at the beginning of your program.
- There is not keyboard input.
- Use statement functions to define the integrand of the integral of Eq. (1) and of $F_b(\omega)$ of Eq. (3).
- Use the Trapezoidal rule to compute the integral of Eq. (1). Use $t(\infty) = 6/b$ and $N = 100$ for the number of grid points on the t-axis.

• For ω, use a `DO` loop to move from 0 to 10 s^{-1} in steps of $\Delta\omega = 10\ s^{-1}/M$, where $M = 300$.

• The numerical results for $F(\omega)$ are to be written to an external data file named `FourierTrNum.dat`.

•The analytic results are to be written to `FourierTrExact.dat`.

• Download the python plotting script `plot.py` from the class website and show your results graphically (in the same plot).

Submission Instructions: Create a gzipped archive named `ws11.tgz` which contains your fortran source code and the `pdf` file `Fourier.pdf` of your plot and email the archive to `ewhart317@gmail.com`. Put `PHYS 317 WS 11` in the subject line.