The Accelerating Universe **Worksheet 13** **6 March 2023**

In this worksheet you will learn how to solve a linear second-order differential equation. For this purpose, the differential equation is expressed as a system of two coupled first-order differential equations.

The Friedman equation is a linear second-order differential equation which predicts the size of our Universe as a function of cosmic time. It is given by

$$\ddot{R}(t) = -\frac{\Omega_{\mathrm{M}}}{2R(t)^2} + \Omega_\Lambda \, R(t)\,, \tag{1}$$

where the input parameters are the matter density, Ω_{M}, and the cosmological constant (or vacuum density, Ω_Λ. Current estimates of the two densities are that the matter density is in the range $0.31-0.33$ and the vacuum density is $0.70-0.72$. The numbers add up to almost exactly 1.0. Lately data has been tending toward a sum of 1.02. The solution of the Friedman equation depends critically on Ω_{M} and Ω_Λ. To solve this equation, one starts at the present where $R = 1$ and $dR/dt = 1$. Solutions of the Friedman equation for different combinations of Ω_{M} and Ω_Λ are shown in Fig. 1.

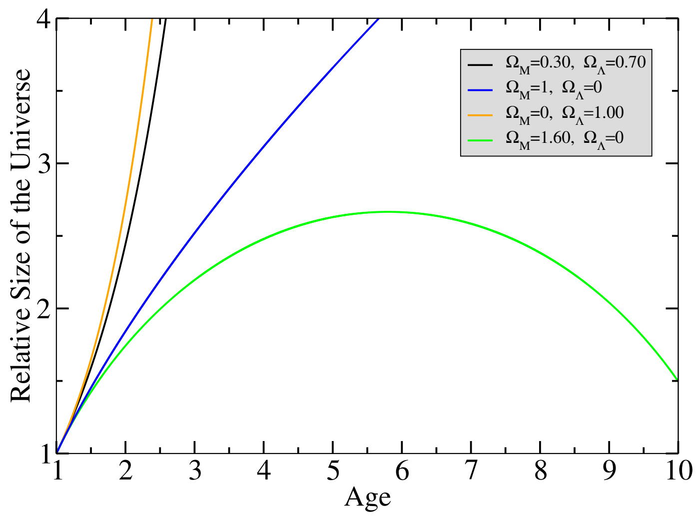

Figure 1: Solutions of the Friedman equation (1) for different values of the matter density Ω_{M} and cosmological constant Ω_Λ.

Task

Write a structured Fortran program which solves Eq. (1) from the present time ($t = 1$) to 10 times the current age ($T = 10$) of the Universe, i.e., $1 \le t \le T = 10$. For this purpose, write Eq. (1) as a system of two coupled first-order differential equations. These equations are then solved using the Euler forward scheme.

Program Design

- Include a short preamble at the beginning of the program.
- The user should be prompted (keyboard input) to enter values for Ω_{M} and Ω_Λ.
- These values are to be written back to screen.

- The Euler forward scheme is to be used to solve the coupled system of differential equations. Use a temporal step size of $\Delta t = 0.001$.
- The (relative) size of the Universe $R(t)$ as a function of time t is to be written to an output file.
- Run the code for the following combinations of matter and vacuum densities and show the results graphically. You may use the python script provided on the class website.

  $(\Omega_{\rm M}, \Omega_\Lambda) = (0.30, 0.70)$, which ware approximately the measured actual values.

  $(\Omega_{\rm M}, \Omega_\Lambda) = (1.0, 0.0)$, which represent the present total density but with no vacuum density contribution.

  $(\Omega_{\rm M}, \Omega_\Lambda) = (0.0, 1.0)$, which represents no matter density and only vacuum density.

  $(\Omega_{\rm M}, \Omega_\Lambda) = (1.60, 0.0)$, which corresponds to a Universe that has 1.6 times the mass density of our Universe but with no vacuum density contribution.

Submission Instructions. Email a gzipped tar file which contains copies of your Fortran source code and pdf plot to `ewhart317@gmail.com`. Put `PHYS 317 WS 13` in the subject line.