Vibrating Pendulum

Worksheet 15
15 March 2023

In this worksheet, you will learn how to numerically solve a 2nd order differential equation. The numerical code will make use of `DO WHILE` *and* `Logical IF` *constructs.*

An experimental system consist of a rigid pendulum (a massless rigid rod and a point mass at its end) whose suspension point is vibrating with frequency Ω along the vertical line. The equation of motion describing such a system is given by

$$\ddot{\phi}(t) + \kappa\lambda\dot{\phi}(t) + \beta\left(1 - \alpha\Omega^2 \sin(\gamma\Omega t)\right)\sin(\phi(t)) = 0\,, \tag{1}$$

where $\phi(t)$ is the angle between the vertical and the pendulum rod. The quantity κ is given by

$$\kappa = \begin{cases} 2 & \text{if } \; t \leq t_{\text{final}}/8 \\ -1.3\,\sin(\phi(t)) & \text{if } \; t_{\text{final}}/8 < t \leq 3\,t_{\text{final}}/4 \\ 2 & \text{if } \; t > 3\,t_{\text{final}}/4 \end{cases} \tag{2}$$

The system's parameters are $\alpha = 0.01$ s^2, $\lambda = 0.01$ s^{-1}, $\beta = 1.001$ s^{-2}, and $\gamma = 0.4$. At time $t = 0$, the pendulum is at $\phi(0) = \pi$, and $\dot{\phi}(0) = 0.5$ s^{-1}.

Tasks

- Re-write Eq. (1) as a system of coupled, first-order differential equations.
- Write a complete Fortran program which solves this system of couples first-order equations numerically for times $0 < t < t_{\text{final}}$, where $t_{\text{final}} = 50$ s. Use a `DO WHILE` to cover $0 < t < t_{\text{final}}$.
- Use a logical `IF` construct for κ.
- Use a value of $dt = 0.001$ s for the incremental time step.
- The value for Ω is keyboard input.
- The program should create an output file which shows the angle ϕ as a function of time.
- Run the program for $\Omega = 75$ Hz and 80 Hz and show the results graphically in the same plot.

Submission Instructions. Email a gzipped tar file containing your Fortran source code and the `pdf` plot to `ewhart317@gmail.com`. Put `PHYS 317 WS 15` in the subject line.