*Purpose: In this worksheet you will learn how to compute the roots of a non-linear equation numerically using Newton's method. The code will make use of FUNCTION sub-programs.*

Given is the following non-linear equation,

$$f(x) = e^x \ \ln(x) - x^2 + 3\,x^3 - 20\,x\,, \qquad 1 \le x \le \pi\,. \tag{1}$$

Task

Write a structured and well commented Fortran program which uses Newton's root-finding method to compute the root of $f(x)$ given by Eq. (1). The function $f(x)$ is shown graphically in the figure below:
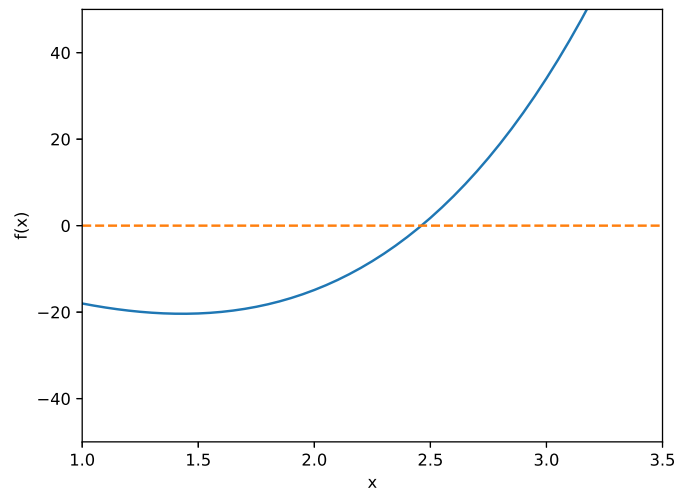
Code Design

- The initial guess value for x (~ 1.5) is keyboard input.

- The maximum number of iterations ($= 20$) is keyboard input.

- The function $f(x)$ and its derivative $f'(x)$ are to be computed in `FUNCTION` sub-programs.

- The root finding iteration scheme is to be stopped if diff $\equiv |x_{i+1} - x_i| \le \epsilon$, where $\epsilon = 10^{-6}$. A possible terminal dialog generated by the code is shown below:

```
Input starting value for x: 1.5
Number of interations (N <=20): 20
it=           0  diff=    9.88233852       x_new=    11.3823385
it=           1  diff=   0.978554726       x_new=    10.4037838
it=           2  diff=   0.987195015       x_new=    9.41658878
it=           3  diff=    1.00552082       x_new=    8.41106796
  .           .          .                           .
  .           .          .                           .
  .           .          .                           .
it=          12  diff=   6.23226166E-04    x_new=    2.46121645
it=          13  diff=   4.76837158E-07    x_new=    2.46121597
```

- A warning message is to be written to standard output if the root of the equation has not been found after the maximum number of iterations has been carried out by the code.

Name your Fortran source code `LastFirst_WS20.f90` and email a copy to `ewhart317@gmail.com`. Put `PHYS 317 WS 20` in the subject line.