Least Squares Method
Dynamic Memory Allocation

Worksheet 21

The purpose of this worksheet is fit a set of experimental data with a mathematical model using the least squares technique. Dynamic memory allocation will be used to define the dimensions of arrays.

In a lab experiment, a group of student measures the magnetic field B (in Tesla) at various distances d (in meter) from a straight wire carrying an electric current. The measured data are shown in Table 1.

Table 1: Experimental data set.

Distance d (m)	Magnetic field B (T)
0.15	0.012
0.25	0.00735
0.33	0.004154
0.48	0.00348
0.55	0.00247
0.62	0.00249
0.81	0.00178
0.89	0.00155
0.91	0.00133
1.0	0.00125
1.13	0.00115
1.25	0.00179
1.37	0.000958
1.49	0.000738
1.55	0.000856
1.62	0.000823
1.77	0.000761
1.85	0.000726
1.89	0.000697
2.02	0.000861
2.08	0.000645
2.25	0.000626
2.17	0.000581
2.42	0.000663
2.46	0.000541
2.69	0.000529
2.70	0.000819
2.73	0.000493
2.96	0.000461
3.09	0.000254

Task

Write a structured and well commented Fortran program that reads the data shown in Table 1 and uses the least-squares technique to fit those data with a best-fit model. A file containing the data shown in this table can be downloaded from the class website.

The best-fit model used for this worksheet uses a straight line, $y = a + b\,x$, to approximate a given number of experimental data, $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, where a and b are given by (see Chapter 4 in the class

textbook):

$$a = \frac{\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} x_i\, y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2}\,, \tag{1}$$

$$b = \frac{\left(n\sum_{i=1}^{n} x_i\, y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2}\,. \tag{2}$$

Numerical Code Design

- Use the `ALLOCATE` statement to let the program dynamically determine how much space is to be allocated for the arrays.
- Use the intrinsic procedure `SUM` to carry out the various summations in Eqs. (1) and (2).
- The screen output produced by the code should be as shown below:

```
Least squares fit to magnetic field data:
 y = a + bx
 a =   . . .
 b =   . . .
```

- The best-fit model data and the original data are to be written to output files and shown graphically. The results are shown in Fig. 7.

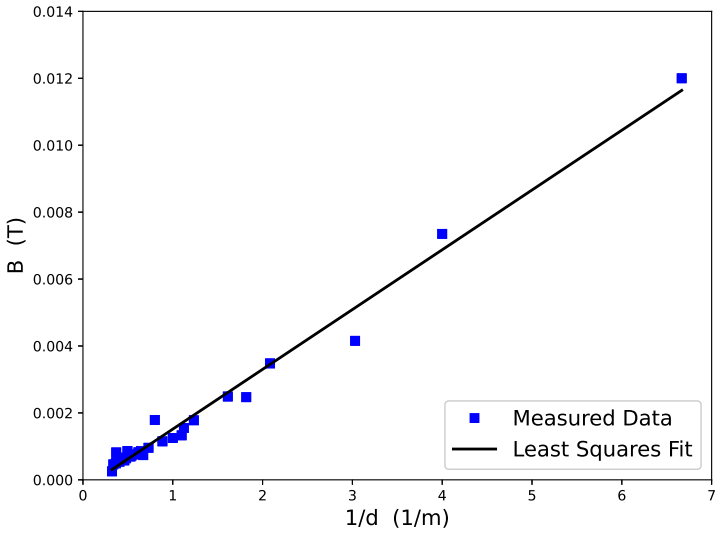

Figure 7: Magnetic field data. The straight line is the best-fit to the experimental data represented by the filled blue squares.

Submission Instructions

Create an gzipped archive named `LastFirst_WS21` which contains your Fortran source code and (`pdf`) plot and email this file to `ewhart317@gmail.com`. Put `PHYS 317 WS 21` in the subject line.