Data Files and Bessel Functions

Worksheet 9
20 February 2023

The purpose of this worksheet is to practice the implicit DO loop construct to read (and then manipulate) data from input files.

Bessel functions play a very in optics and quantum mechanics. The Bessel functions of the first kind $J_n(x)$ are defined as the solution to the Bessel differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right) y = 0\,. \tag{1}$$

They are sometimes also referred to as cylinder functions or cylindrical harmonics. Figure 1 shows $J_n(x)$ for $n = 0, 1, 2, \ldots, 5$. The numerical values of $J_0(x), \ldots, J_6(x)$ for $0 \leq x \leq 20$ are stored in an archive named

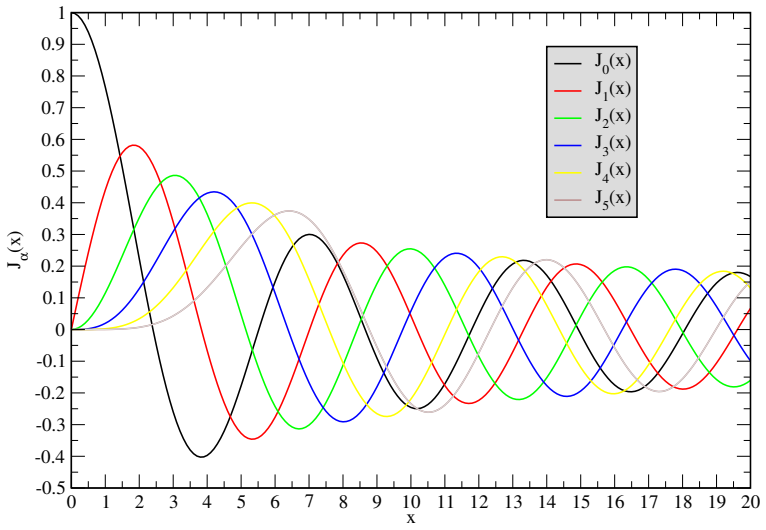

Figure 1: The n^{th}-order Bessel functions of the first kind $J_n(x)$ for $0 \leq x \leq 20$.

`bessel_functions.tar.gz`. on the class website.

Tasks

• Download `bessel_functions.tar.gz` from the class website.

• Use `tar -tzvf bessel_functions.tar.gz` to list the content of the archive and then extract the content with `tar -xzvf bessel_functions.tar.gz`. This will create a sub-directory on your machine named `Bessel_J0_J6` which contains the Bessel functions $J_n(x)$ for x values in the range $0 \leq x \leq 20$ ($n = 0, 1, 2, \ldots, 6$).

• Use the commands `more` and `tail` to display the first/last few lines of each data file.

• Write a structured and well-commented Fortran program which reads the numerical values of each data file (i.e., x and $J_n(x)$), computes $J_n(x)^2$ for $0 \leq x \leq 20$, and writes $J_n(x)^2$ as a function of x to an output

file. Note that seven different output files must be created, since $n = 0, 1, 2, \ldots, 6$. Use the implicit `DO` loop construct to read the data from each input file. The syntax will be something like this:

```
      DO
      READ (100, *, end=20) x, J_x
      WRITE(200, *) x, J_x**2
      END DO
   20 CONTINUE
```

Below are some sample results for $J_0(x)^2$ for selected x values:

```
     x               J_0(x)**2
 0.00000000       1.00000000
 1.00000000       0.585527539
 2.00000000       5.01270816E-02
 3.00000000       6.76270276E-02
 4.00000000       0.157727972
 5.00000000       3.15406136E-02
 6.00000000       2.26939935E-02
 7.00000000       9.00475606E-02
 8.00000000       2.94640008E-02
 9.00000000       8.16016085E-03
10.0000000        6.04844019E-02
20.0000000        2.78976895E-02
```

Submission Instructions: Rename your Fortran source code to `LastFirst_WS9.f90` and email it to `ewhart317@gmail.com`. Put `PHYS 317 WS 9` in the subject line.