SAN DIEGO STATE UNIVERSITY
Department of Physics
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Physics 570–Relativity Fall 2017

Assignment 2

Due September 15, 2017

Problem 1. In a 3-dimensional space with coordinates x^1, x^2, x^3, expand (i.e., write out) the expression $S = g_{ij}\, x^i\, x^j$.

Problem 2. Express $b^{ij}\, y_i\, y_j\, z_k$ in terms of x-variables, if $y_i = c_{ij}\, x^j$, $z_k = d_{kl}\, x^l$, and $b^{ij}\, c_{ik} = \delta^j{}_k$.

Problem 3. For an 5-dimensional flat space, evaluate (a) $\delta^i{}_i$, (b) $\delta^i{}_j\, \delta^j{}_i$, (c) $\delta^i{}_j\, \delta^j{}_k\, c_i{}^k$ where $(c_i{}^k)$ is the 5×5 identity square matrix with ones on the diagonal and zeros elsewhere.

Problem 4. Write out the expression $R_\mu{}^{\nu\lambda\mu}$ in full in a 4-dimensional space. Which are free and which are dummy indices? How many summations are there?

Problem 5. Calculate all components of the metric tensor in 3D Euclidean space ($ds^2 = dx^2 + dy^2 + dz^2$) for the coordinates $u = x + 2y$, $v = x - y$, and $w = z$.

Problem 6. The basis representation of a $\binom{0}{2}$ tensor is given by $T = T_{\alpha\beta}\, b^{\alpha\beta}$, where $b^{\alpha\beta}$ denote a set of basis vectors, which are to be determined. Show that they are given by

$$b^{\alpha\beta} = \tilde{\omega}^{\,\alpha}(\vec{e}_\mu)\, \tilde{\omega}^{\,\beta}(\vec{e}_\nu)\,.$$