Mass Formula of Atomic Nuclei — Homework 3
Due 15 February 2023

The Bethe-Weizsäcker mass formula expresses the mass of an atomic nucleus, M, as a function of proton number Z and nucleon (baryon) number A as follows:

$$M(Z,A) \;=\; ZM_{\rm H} + (A-Z)M_{\rm n} - a_{\rm V}A + a_{\rm S}A^{2/3} + a_{\rm C}Z^2A^{-1/3} + a_{\rm A}(Z-A/2)^2A^{-1}\,. \tag{1}$$

The quantities $a_{\rm V} = 15.8$ MeV, $a_{\rm S} = 18.3$ MeV, $a_{\rm C} = 0.71$ MeV, and $a_{\rm A} = 92.7$ MeV are experimentally determined constants. (Use $M_{\rm n} = 939.57$ MeV and $M_{\rm H} = 938.78$ MeV for the mass of a neutron and the mass of a hydrogen atom, respectively.) The energy per baryon (binding energy), $E_B(Z,A)$, of an atomic nucleus follows from Eq. (1) as

$$\begin{aligned} E_B(Z,A) &\equiv \left(M(Z,A) - ZM_{\rm H} - (A-Z)M_{\rm n}\right)\,A^{-1} && (2)\\ &= -a_{\rm V} + a_{\rm S}A^{-1/3} + a_{\rm C}Z^2A^{-4/3} + a_{\rm A}(Z-A/2)^2A^{-2} && (3)\\ &\equiv E_{\rm Volume} + E_{\rm Surface} + E_{\rm Coulomb} + E_{\rm Asymmetry}\,, && (4)\end{aligned}$$

where

$$\begin{aligned} E_{\rm Volume} &= -a_{\rm V}\,, && (5)\\ E_{\rm Surface} &= a_{\rm S}A^{-1/3}\,, && (6)\\ E_{\rm Coulomb} &= a_{\rm C}Z^2A^{-4/3}\,, && (7)\\ E_{\rm Asymmetry} &= a_{\rm A}(Z-A/2)^2A^{-2}\,, && (8)\end{aligned}$$

denote volume energy, surface energy, Coulomb energy, and asymmetry energy of a given atomic nucleus.

Task

Write a structured Fortran 90 program which computes $E_{\rm Volume}$, $E_{\rm Surface}$, $E_{\rm Coulomb}$, $E_{\rm Asymmetry}$, $M(Z,A)$, and $E_B(Z,A)$ of an atomic nucleus with a given proton number (Z) and baryon number (A).

Program Design

- Your program must contain a preamble.
- Your program must be well commented.
- Z, A, N must be defined as integers.
- The proton number, Z, and baryon number, A, are keyboard input.
- Use a `DO` loop which prompts the user to enter the values for Z and A for 7 different atomic nuclei, one at a time, via the keyboard. The code then calculates the properties of an atomic nucleus for the given Z and A values, computes and outputs the properties of this atomic nucleus (see below), and then prompts the user to enter the A and Z values of the next nucleus. This process is repeated until the input for the $7^{\rm th}$ nucleus has been made. The screen output for a nucleus like $^{244}_{94}$Pu should be as shown here:

```
 Input proton number (Z):
94
 Input baryon number (A):
244
 N=         150  (Number of neutrons)
 Z=          94  (Number of protons)
 A=         244  (Baryon number)
 M(Z,A)=   227342.016       MeV
 E_B=  -7.53611803       MeV
 E_volume=  -15.8000002     MeV
```

```
 E_surface=   2.92856193     MeV
 E_coulomb=   4.11459875     MeV
 E_asymmetry=   1.22072017     MeV
```

Perform the following numerical investigations with your code

1. Use this program to determine the masses and binding energies of the following atomic nuclei in a single run:[1]

$^{70}_{32}$Ge, $^{179}_{72}$Hf, $^{38}_{18}$Ar, $^{56}_{26}$Fe, $^{40}_{20}$Ca, $^{235}_{92}$U, $^{238}_{92}$U.

2. Copy the results of each calculation to a text file named `NuclearData.dat`.

HOMEWORK SUBMISSION INSTRUCTIONS

1. Create a sub-directory `LastFirst_HW3`.

2. Copy the source code of your Fortran 90 program and the data file to `LastFirst_HW3`.

3. Create a gzipped archive file of your homework by typing
`tar -czvf LastFirst.tgz LastFirst_HW3/.`

4. Email the archive file `LastFirst.tgz` to `ewhart317@gmail.com`. Put `PHYS 317 HW 3` in the subject line.

[1] We use the notation $^{A}_{Z}$X for an atomic nucleus of type X, where $A = N + Z$ is the number of nucleons (baryon number), N the number of neutrons, and Z the number of protons of nucleus X.