Nested DO Loops

Worksheet 6
8 February 2023

The purpose of this worksheet is to use the nested DO loop construct to compute double and triple sums as well as a product of functions.

Task

Write a structured Fortran 90 program which computes and prints out (standard output) the results of

$$A(N) = \sum_{i=1}^{N} \sum_{j=1}^{i} j^{-2}\,(i+1)^{-2}\,, \tag{1}$$

$$B(N) \equiv \prod_{k=1}^{N-1} \sin\left(k\pi/N\right)\,, \tag{2}$$

$$C(N) = \left(32\left(\sum_{k=0}^{N} \frac{k+1}{(2k+1)^2\,(2k+3)} - \frac{1}{8}\right)\right)^{1/2} \tag{3}$$

$$T(N) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{1}{(i\;j\;k)^2}\,, \tag{4}$$

for $N = 10\,(10)\,100$. The results for $A(N)$, $B(N)$, $C(N)$, and $T(N)$ are to be compared with the analytic results given by $A(\infty) = \pi^4/120 = 0.811742$, $B(N) = 2^{1-N}\,N$, $C(\infty) = \pi$, and $T(\infty) = \pi^6/216 = 4.45088$. Note: The symbol $\prod$ in Eq. (2) denotes the $\prod$-product. It is like the summation symbol $\sum$ but rather than addition, its operation is multiplication. For example, $\prod_{k=1}^{5} f_k = f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5$.

The (unformatted) terminal output produced by your code should be as shown below:

```
N=  10
A=  0.672680 A(analytic)=  0.811742
B=  1.953127E-02 B(analytic)=  1.953125E-02
C=  3.08448 C(analytic)=  3.14159
T=  3.72220 T(analytic)=  4.45087

N=  20
A=  0.736346 A(analytic)=  0.811742
B=  3.814708E-05 B(analytic)=  3.814697E-05
C=  3.11149 C(analytic)=  3.14159
T=  4.06660 T(analytic)=  4.45087

.
.
.

N=  100
A=  0.795585 A(analytic)=  0.811742
B=  1.577734E-28 B(analytic)=  1.577722E-28
C=  3.13530 C(analytic)=  3.14159
T=  4.37059 T(analytic)=  4.45087
```

Program design

• There is no keyboard input.

• Use `pi = ACOS(-1.0)` to define π.

• Use an outer `DO` loop which determines the value of N. The values of N run from 10 to 100 in steps of 10 (i.e., 10 (10) 100).

• Inside this outer `DO` loop are four nested `DO` loop constructs which, for a given N value, compute the expressions shown in Eqs. (1) and (4).

• For each N value, the results for $A(N)$, $B(N)$, $C(N)$, and $T(N)$ are to be written to standard output.

Submission Instructions: Rename your Fortran source to `LastFirst_WS6.f90` and email this file to `ewhart317@gmail.com`. Put `PHYS 317 WS 6` in the subject line.