Numerical Integration I
The Trapezoidal Rule

Worksheet 7
13 February 2023

Purpose: In this worksheet you will learn how two integrate a continuous function numerically using the Trapezoidal rule.

Let $f(x)$ denote a continuous function and $h = (b-a)/N$ (N is an integer). The Trapezoidal rule is give by:[13]

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f_0 + 2\sum_{k=1}^{N-1} f_k + f_N\right), \qquad N \text{ is even or odd}, \tag{1}$$

Task

Write a structured and well-commented Fortran program that uses the Trapezoidal rule to compute the integral

$$W \equiv \int_a^b x\sqrt{x}\sinh(x)\ \mathrm{e}^{-\sqrt{x}\cos(x)-x^2\sin(x^2)}\ \left(1.0+0.5x+0.2x^2+0.1x^4\right)^{-1}\ dx \tag{2}$$

for ($a = 0$, $b = 1.0$), ($a = 0$, $b = 1.60$), and ($a = 1.0$, $b = 1.6$). The number of grid points, N, to be used in Eq. (1) to compute the integral of Eq. (2) should range from 2 to 100, in increments of 2.

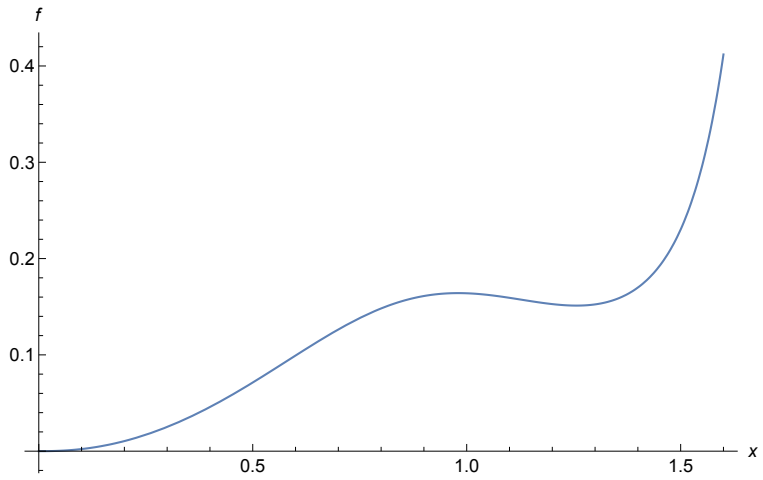

Figure 7: Graphical illustration of the integrand of Eq. (2).

Code Design

- Your code must contain a preamble.
- The values of the integration limits a and b are keyboard input.
- The number of grid points, N, used to compute the integral in Eq. (2) is determined by a `DO` loop. The N-values range from 2 (2) 100.
- Use a statement function to define the integrand in Eq. (2).
- The terminal output generated by your program for $a = 0$ and $b = 1.0$ should be as follows:

```
N=           2 W_Trap=   7.66319335E-02 a=   0.00000000     b=   1.00000000
```

[13] See Section 4.3 in Class Textbook.

```
N=           4 W_Trap=   7.71269202E-02 a=   0.00000000     b=   1.00000000
N=           6 W_Trap=   7.71715343E-02 a=   0.00000000     b=   1.00000000
.
.
.
N=          96 W_Trap=   7.72050917E-02 a=   0.00000000     b=   1.00000000
N=          98 W_Trap=   7.72050917E-02 a=   0.00000000     b=   1.00000000
N=         100 W_Trap=   7.72050992E-02 a=   0.00000000     b=   1.00000000
```

For $a = 0$ and $b = 1.60$, the terminal output has the form:

```
N=           2 W_Trap=  0.283346236     a=   0.00000000     b=   1.60000002
N=           4 W_Trap=  0.221091896     a=   0.00000000     b=   1.60000002
N=           6 W_Trap=  0.205621883     a=   0.00000000     b=   1.60000002
.
.
.
N=          96 W_Trap=  0.190015063     a=   0.00000000     b=   1.60000002
N=          98 W_Trap=  0.190012231     a=   0.00000000     b=   1.60000002
N=         100 W_Trap=  0.190009519     a=   0.00000000     b=   1.60000002
```

For $a = 1$ and $b = 1.60$, the terminal output is give by:

```
N=           2 W_Trap=  0.132111773     a=   1.00000000     b=   1.60000002
N=           4 W_Trap=  0.118213646     a=   1.00000000     b=   1.60000002
N=           6 W_Trap=  0.115244508     a=   1.00000000     b=   1.60000002
.
.
.
N=          96 W_Trap=  0.112748884     a=   1.00000000     b=   1.60000002
N=          98 W_Trap=  0.112748496     a=   1.00000000     b=   1.60000002
N=         100 W_Trap=  0.112748086     a=   1.00000000     b=   1.60000002
```

Note the dependence of the numerical results on the number of grid points used to compute the integral.

Rename your Fortran source code to `LastFirst_WS7.f90` and email a copy of the code to `ewhart317@gmail.com`. Put `PHYS 317 WS 7` in the subject line.