SAN DIEGO STATE UNIVERSITY
Department of Physics
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Physics 570–Relativity Fall 2017

Assignment 1

Due September 8, 2017

1. Read Chapters 1 (Gravitational Physics) and 3 (Space, Time, and Gravity in Newtonian Physics) in J. B. Hartle's book "Gravity: An Introduction to Einstein's General Relativity" (Addison Wesley 2003).

2. Problem 2 on page 45.[1]
Hint: Let $(t, x))$ be the coordinates of an inertial frame and (t', x') the coordinates of a frame accelerating along the x-axis with acceleration g.

3. Problem 4 on page 46.
Hint: Solve Poisson's equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\,\frac{dU}{dr}\right) = 4\pi G\mu\,, \tag{1}$$

where $\mu = 3M/4\pi R^3$=const denotes the mass density of the sphere. Instead of integrating Eq. (1), argue that the general solution must be given by $U(r) = Ar^2+Br+C$, where A, B, and C are constants (which you need to determine).

4. Using Einstein's summation convention and $\partial x^i/\partial x^j = \delta^i{}_j$, calculate

$$\frac{\partial}{\partial x^k}\left(a_{ij}x^i x^j\right) \quad \text{and} \quad \frac{\partial^2}{\partial x^k \partial x^l}\left(a_{ij}x^i x^j\right)\,,$$

where $a_{ij} = a_{ji}$ are constants.

5. Listed below are several mathematical relations which make use of Einstein's summation convention. Which of these relations are correct and which are incorrect? Justify your answers mathematically. (Note: $a_{ij} \neq a_{ji}$.)

$$a_{ij}\,(x^i + y^j) \;=\; a_{ij}\,x^i + a_{ij}\,y^j\,, \tag{2}$$

$$a_{ij}\,x^i\,y^j \;=\; a_{ij}\,y^i\,x^j\,, \tag{3}$$

[1] The page numbers refer to J. B. Hartle's book "Gravity: An Introduction to Einstein's General Relativity" (Addison Wesley 2003).

$$(a_{ij} + a_{ji})\, x^i\, y^j = 2\, a_{ij}\, x^i\, y^j\,, \tag{4}$$

$$a_{ij}\, (x^j + y^j) = a_{ij}\, x^j + a_{ij}\, y^j\,, \tag{5}$$

$$a_{ij}\, x^i\, y^j = a_{ij}\, y^j\, x^i\,, \tag{6}$$

$$a_{ij}\, x^i\, x^j = a_{ji}\, x^i\, x^j\,, \tag{7}$$

$$(a_{ij} + a_{ji})\, x^i\, x^j = 2\, a_{ij}\, x^i\, x^j\,, \tag{8}$$

$$(a_{ij} - a_{ji})\, x^i\, x^j = 0\,. \tag{9}$$